OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2000
Midterm Exam #2

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:

Extend the set

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

to form a basis for the set with all 2×2 matrices with real coefficients.

Then, determine the representation of

$$x = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

with respect to the basis defined.

Problem 2:

Consider the linear operator

$$A = \begin{bmatrix} 1 & 4 & 7 & 3 \\ 2 & 0 & 2 & 1 \\ 3 & 4 & 9 & 4 \end{bmatrix},$$

determine its rank and nullity, then find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 3:
Show that if the set $\{u, v, w\}$ is linearly independent, then so is the set $\{u+v, v+w, w+u\}$.

Problem 4:

Show if the following sets

$$\begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ 2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \\ 2 \end{bmatrix}$$

span the same subspace V of $\left(\Re^4, \Re\right)$.

<u>Problem 5</u>:

Let

$$V^{\perp} = \left\{ x \mid x = \boldsymbol{a}\begin{bmatrix}1\\1\\2\\0\end{bmatrix} + \boldsymbol{b}\begin{bmatrix}-1\\5\\6\\2\end{bmatrix} + \boldsymbol{g}\begin{bmatrix}-1\\2\\2\\1\end{bmatrix}, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{g} \in \Re \right\},$$

determine the original space, *V*, and find an orthogonal basis for *V*. For $x = \begin{bmatrix}-2 & 4 & 4 & 2\end{bmatrix}^T$, find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in V$, and $x_2 \in V^{\perp}$.